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Implicit Quotas

Roland G. Fryer, Jr.

ABSTRACT

Employment or admission “goals” are often preferred to affirmative action as a way of obtaining diversity. By constructing a simple model of employer-auditor interaction, I show that when an auditor has imperfect information regarding employers’ proclivities to discriminate and the fraction of qualified minorities in each employer’s applicant pool, goals are synonymous with quotas. Technically speaking, any equilibrium of the auditing game involves a nonempty set of employers who hire so that they do not trigger an audit by rejecting qualified nonminorities, hiring unqualified minorities, or both. Further, under some assumptions, explicit quotas (those mandated by an auditor) are more efficient than implicit quotas (goals settled on in equilibrium by employers wishing to avoid an audit).

Since President Nixon was here in my job, America has used goals and timetables to preserve opportunity and to prevent discrimination, to urge businesses to set higher expectations for themselves and to realize those expectations. But we did not and we will not use rigid quotas to mandate outcomes. [President William J. Clinton, July 19, 1995]

I am for Affirmative Action, as I describe it, but not for quotas or preferences. [President George W. Bush, April 2, 2000]

We do not think it matters whether a government hiring program imposes hard quotas, soft quotas, or goals. Any of these techniques induces an employer to hire with an eye toward meeting the numerical target. [Judge Laurence Silberman, *Lutheran Church–Missouri Synod v. FCC*, 141 F.3d 344 (D.C. Cir. 1998)]

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1. INTRODUCTION

In its 40-year history, there have been many popular misconceptions about affirmative action, including the following: (1) the only way to create a color-blind society is to adopt color-blind means, (2) affirmative action may have been necessary 30 years ago, but the playing field is fairly level now, (3) the public does not support affirmative action anymore, (4) a large percentage of white workers will lose out if affirmative action is continued, and (5) goals and timelines are better than rigid affirmative action quotas. The first four misconceptions have been shown to be more myth than fact (Fryer and Loury 2005; Fryer, Loury, and Yuret 2008; Loury 1977). Yet goals are thought to be good-faith efforts on the part of noble employers, whereas quotas are envisioned as rigid racial diversity requirements that often result in the hiring of incompetent minorities. In its landmark decision *Regents of the University of California v. Bakke* (438 U.S. 265 [1978]), the Supreme Court ruled such inflexible quotas to be unconstitutional, while it upheld the use of soft quotas, or goals, in *Fullilove v. Klutznick* (448 U.S. 448 [1980]). Understanding the relationship between these amorphous terms is the subject of this paper.

To get beneath the terminology, I develop a model of employer-auditor interaction that involves imperfect auditing of an employer's hiring practices. Employers differ in their proclivities to discriminate and in the fraction of qualified minorities who apply for positions in their firms. After observing its own type (desire to discriminate and applicant pool), each employer hires a ratio of minorities to nonminorities. Thus, if an employer hires a small share of minorities, it implies one of two things: the employer is either a discriminator who rejected some qualified minority candidates or a nondiscriminator who had a small fraction of qualified minorities apply. I assume that an outside auditor cannot distinguish perfectly between these states—even after an audit. The auditor observes each employer's workforce and decides whether to conduct an audit. So, in an effort to eliminate discrimination, the auditor will mis-

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takenly punish employers who did not discriminate while others (who actually did discriminate) go undetected. It is this informational asymmetry that gives employers incentives to alter their hiring ratio so as not to induce an audit.

The results of the simple auditing model are illuminating: all equilibria exhibit an implicit-quota property. That is, a nonempty set of employers (both those who are inclined to discriminate and those who are not) are willing to alter their behavior to avoid an audit, since there is a positive probability that the auditor makes a mistake and the penalty is strictly positive. I use the modifier “implicit” for a particular reason. If we were deriving explicit quotas, this would be represented by the government announcing a desired ratio of minorities to nonminorities and (assuming the penalty for deviating is sufficiently large) employers strictly adhering to this ratio, which has been ruled unconstitutional (*Regents of the University of California v. Bakke*, 438 U.S. 265). In contrast, implicit quotas are those that the employers themselves set, in equilibrium, as an optimal response to imperfect auditing. Thus, the quotes that I began with and the rhetoric from both political parties that supports goals but not quotas have no content.

The lesson is straightforward. If a regulator is interested in enforcing antidiscrimination laws, then goals are quotas when an auditor has imperfect information regarding employers’ desires to discriminate and the fraction of qualified minorities who applied to each firm. Under some assumptions, explicit quotas are more efficient than implicit quotas.¹

This model, although applied here to auditing in the labor market (where quotas are most controversial), can naturally be applied to auditing environments involving tax evasion, teacher accountability, and antitrust enforcement.²

1. Imperfect information is the crucial assumption. Without it, goals and quotas can be quite different objects. But, in practice, the information auditors have on employers’ discriminatory intentions or the quality of their applicant pools is far from perfect.

2. This paper is related to the well-developed literatures on employment discrimination and tax compliance. There is a relatively large literature on employment discrimination. The two main theories are given by Becker (1957) and Arrow (1973). Becker (1957) provides a taste-based theory of discrimination. In this theory, agents discriminate because there exist nonpecuniary psychic costs to interaction with minorities. Thus, in this model, agents are willing to forgo profits or earn lower wages to ensure segregation. Arrow (1973) discusses a model of statistical discrimination. This model shows that employers can (rationally) discriminate against a group even when they are ex ante identical. Independent of the underlying theory of discrimination, it manifests itself in my model by an employer hiring fewer minorities than they otherwise would. A regulator, then, may want to break such equilibria. This auditing problem is similar to the extensive literature on tax com-

The paper proceeds as follows: Section 2 provides a concise but relatively informal verbal description of an auditing model with imperfect information, and it constructs a numerical example that illustrates the main results. Section 3 concludes. Appendix A contains the formal model along with technical proofs of all the results discussed in Section 2. Appendix B provides additional results from the model.

2. A MODEL OF IMPERFECT AUDITING

Let there be a continuum of workers and a continuum of employers. Workers belong to one of two groups: minorities or nonminorities. There are also two types of employers: some are biased against minorities, while the others are unbiased. There is a set of auditors. Before the start of the game, the government chooses a penalty to be enforced on employers who discriminate against minorities in their hiring practices.

Nature moves first and assigns a two-dimensional type to each employer: whether it is biased and the quality of its applicant pool. The latter is a number on the positive real line, distributed according to a smooth and continuous cumulative distribution function. One can think of this number as the profit-maximizing (absent discriminatory taste) ratio of minorities to nonminorities in an employer's applicant pool.³ This formulation is flexible enough to allow for different distributions of effort, investment, talent, geography, or other factors that might change the profit-maximizing ratio of minorities to nonminorities an employer wants to hire.

Next, employers observe their two-dimensional private types and make hiring decisions. The following provides a formal definition of discrimination.

pliance (see Andreoni, Erard, and Feinstein [1998] for an overview). Surprisingly, however, the literature on imperfect auditing is small (Lawarree and Van Audenrode 1992; Kofman and Lawarree 1993; Bardsley 1996; and, implicitly, Erard and Feinstein 1994). The closest to my approach is Bardsley (1996), who has a similar model of imperfect auditing. The key difference involves the auditors' payoffs.

3. Ideally, one would want to endogenize the employer's state and allow employers to make investments to increase their likelihood of being in a "good" state, using the monotonic likelihood ratio property. I do not model these initial investments by the employers since they are not observable by the auditor. If it helps to fix ideas, one can assume that the "lottery" of states is determined by investment (that is, recruitment) activities of firms outside my model. However, if the function that maps recruiting initiatives to applicant pools is not deterministic (that is, intense minority recruiting need not always result in a minority-rich applicant pool), then the exogenous determination of states is without further loss of generality.

Definition 1. A firm is said to discriminate if it hires a ratio of minorities to nonminorities that is strictly less than its profit-maximizing ratio (not including its possible discriminatory taste).

By definition, unbiased employers hire the profit-maximizing ratio of minorities to nonminorities, absent regulation. I further assume that biased employers, absent regulation, hire a strictly lower ratio. In the language of Becker (1957), one can think of this difference as a discrimination coefficient, while in statistical or cognitive discrimination models, it may capture the lower share of blacks hired owing to negative stereotypes (Arrow 1973) or coarser categories (Fryer and Jackson 2008). An auditor, after observing each employer's hiring decision (not its type or applicant pool), makes a dichotomous audit decision: audit or not. If the auditor decides to conduct an audit, she makes a correct assessment of the employer with probability greater than a half, and she makes a mistake with the complementary probability. After the audit, the auditor decides whether to issue the fine.⁴ It is important to emphasize that discrimination here is thought to be one-sided: a regulator is auditing hiring practices to lessen discrimination against minorities that exists absent regulation. In a more elaborate model, one can add penalties for overshooting and discriminating against nonminorities.

An employer's payoffs are represented by a single-peaked function that reaches its maximum when the employer hires its optimal ratio of minorities to nonminorities, taking into account its applicant pool and possible discriminatory preferences. Auditors receive a positive payoff if they punish a discriminating employer and suffer a cost if they do not fine a discriminating employer or mistakenly fine a nondiscriminator.⁵ The payoff to an auditor for not punishing a nondiscriminator is normalized to zero.

It is assumed that auditors are interested only in finding and punishing employers who discriminate against minorities. They do not care about employers' (possibly) biased preferences toward minorities, as long as they do not use discriminatory hiring practices. To keep things simple, I assume that all payoff-relevant parameters are exogenously given.

4. Realistically, the punishment should be proportional to the level of discrimination. Adding more elaborate penalty functions will make interesting changes in the qualitative properties of the equilibria. But if the penalty is strictly positive for all discriminatory acts, my main result holds.

5. Assuming that the auditor receives negative utility from punishing a nondiscriminator is equivalent to the limited-liability assumption in the tax literature.

2.1. Equilibrium

To solve the model, I focus on pure-strategy equilibria in which each agent makes a deterministic choice and all individuals of the same type make the same choice.⁶ The solution concept for the auditing model, per usual for signaling games, is a perfect Bayesian equilibrium. Intuitively, a perfect Bayesian equilibrium is a set of strategies and beliefs such that, at any stage of the game, strategies are optimal given the beliefs, and beliefs are obtained from equilibrium strategies and observed actions using Bayes's rule. In what follows, I describe existential results for two possible sets of equilibria: separating and semiseparating. All formal statements of the propositions, along with their proofs, can be found in Appendix A. Appendix B treats the possibility of pooling, non-monotonic, and mixed-strategy equilibria.

2.1.1. *Separating Equilibrium.* For (standard) monotonic signaling models, in a fully separating equilibrium, an agent of each type chooses a unique action, and each type is correctly identified in equilibrium. In this (slightly nonstandard) nonmonotonic signaling game, full separation is ruled out a priori, owing to continuous types and strategy spaces. In particular, for every hiring ratio there exist two applicant pools—one richer in qualified minorities than the other—such that a biased employer with the better pool will hire the same number of minorities as an unbiased employer with fewer qualified minorities among her applicants. Consider the following (slightly perturbed) definition of a separating equilibrium.

Definition 2. In any pseudoseparating equilibrium, each employer hires its profit-maximizing ratio of minorities to nonminorities.

The first result (proposition 1, Appendix A) highlights the fact that no pseudoseparating equilibrium exists. Any effort on the auditor's part to find and punish discriminators will necessarily yield an implicit quota (employers will hire more minorities than they would in their profit-maximizing workforce). This is the main theme of the paper. The surprising

6. A strategy for an employer is an assignment function that maps its private type to a ratio of minorities to nonminorities hired. A strategy for an auditor is a function that maps an employer's observed ratio of minorities to nonminorities hired to an audit decision. To begin, I restrict attention to monotonic strategies for the auditor (that is, cutoffs), in which she audits any employer with an observed ratio of minorities to nonminorities below the cutoff and does not audit any employer with an observed ratio above it. This is without loss of generality when the auditor uses pure strategies (see Appendix B).

part of this result is that biased and unbiased employers alike may adhere to the implicit quota.

Some believe that quota-like hiring from employers would be an easier pill to swallow if the market were not accounting for tastes, and the threat of an audit simply forced biased employers to hire workforce ratios that were equivalent to what unbiased employers would optimally hire, conditional on the same applicant pool.⁷ This is possible if the auditor has perfect information. However, some may find it disturbing that, given the auditor's lack of information, even unbiased employers are willing to alter their hiring ratios so as not to induce an audit, especially when (as proposition 1 proves) this behavior is inevitable, for at least some employers.

2.1.2. Semiseparating Equilibria. There are two types of semiseparating equilibria, which I label "marginal" and "inclusive." The distinction between them hinges on what types of employers choose to pool on the implicit quota. Marginal equilibria require that only marginal employers (employers whose profit-maximizing hiring ratios are relatively close to the implicit quota) adhere to the implicit quota. In this type of equilibrium, employers with applicant pools that have very few qualified minorities refuse to alter their hiring ratios enough to avoid an audit, because the profit loss in doing so is large relative to the expected cost of being audited. They simply incur the expected cost. As the fine for being deemed a discriminator gets large, fewer employers will risk the penalty, and inclusive equilibria will result. In an inclusive semiseparating equilibrium, all employers (whether or not they are biased) who face a profit-maximizing hiring ratio below the implicit quota will alter their behavior and hire right up to the implicit quota. Note that marginal equilibria are the only equilibria for which audits occur in equilibrium. In this sense, one may find them more appealing and empirically relevant.

Whether marginal or inclusive equilibria result depends solely on the magnitude of the penalty for discriminating. If the expected penalties are relatively small, marginal equilibria exist (proposition 2, Appendix A); if the expected penalties are large, inclusive equilibria result (proposition 3, Appendix A).

The technical conditions to ensure a marginal equilibrium require that all employers with relatively small fractions of qualified minorities in their applicant pools hire their profit-maximizing workforce. This puts an upper bound on the penalty that can be imposed in equilibrium. The

7. However, others believe that not accounting for market tastes is a mistake, even if it means that some groups endure discriminatory treatment (see Epstein 1992).

conditions also ensure that there is always a set of positive measure of employers who alter their profit-maximizing workforces by “jumping up” to the implicit quota. And, given this behavior from employers, the auditor does not find it worthwhile to audit at that quota. The conditions for inclusive equilibrium ensure that the expected costs of being audited are sufficiently high to dissuade potential deviators, and the implicit quota is high enough to minimize the amount of discrimination in equilibrium, so that the auditor does not find it optimal to audit the employers who pile up at the quota even though she knows that some of them are discriminating.

A simple numerical example illustrates many of the points stressed thus far. Assume that the auditor believes an employer who hires a certain ratio of minorities to nonminorities to be a discriminator with a probability of one minus the hiring ratio. Further, assume that the auditor receives a payoff of one if she correctly punishes a discriminator and incurs a penalty of one if she mistakenly fines a nondiscriminating employer. Finally, assume that the probability of making a correct assessment is 80 percent and the costs of conducting an audit are fixed at $\frac{7}{10}$. Under these circumstances, the auditor will find it optimal to audit all employers who hire less than 50 percent minorities (see Appendix A for derivation). The employer’s utility function reaches a maximum of zero when it hires its profit-maximizing ratio. Whenever an employer deviates from this ratio, he receives a penalty equal to the squared deviation. Also, suppose that discriminating employers will hire only half the number of minorities an otherwise equal, unbiased employer would hire. Under these assumptions, a marginal semiseparating equilibrium will exist if the fine is set to $\frac{5}{64}$. Then, all employers, whether they are discriminators or not, with optimal hiring ratios between $\frac{3}{8}$ and $\frac{1}{2}$ will alter their behavior and hire 50 percent minorities. Those employers who would hire less than $\frac{3}{8}$ will not change their hiring decisions but rather incur the expected costs of being audited. Employers who would hire more than 50 percent anyway will also not deviate, as they will not be audited in equilibrium. If, however, the fine is set higher than $\frac{5}{64}$, then an inclusive semiseparating equilibrium exists. That is, all employers who would otherwise hire a fraction of minorities smaller than $\frac{1}{2}$ will now hire exactly up to this line to avoid the possibility of being fined by the auditor. All other employers will, again, not deviate from their optimal hiring ratios.

2.2. The Multiplicity Problem

In typical signaling models, one is plagued with the multiplicity of equilibria owing to the freedom associated with out-of-equilibrium beliefs in standard solution concepts. For example, suppose that we have an inclusive equilibrium with a lower-bound hiring ratio, and the auditor happens to observe an employer who hires below that ratio. In this case, a perfect Bayesian equilibrium does not specify the auditors' inferences; thus, it is theoretically plausible that an auditor will believe that any deviations below that ratio certainly indicate discriminators. She could just as easily believe that they are nondiscriminators. She is free to choose. As a result of the lackadaisical requirements on out-of-equilibrium beliefs imposed by Bayesian perfection, we have a continuum of potential equilibria (propositions 4 and 5, Appendix A). For instance, we know that in any equilibrium the auditor does not want to audit employers who pile up on the implicit quota. Well, there is a continuum of possible implicit quotas above which an auditor is indifferent. With out-of-equilibrium beliefs that anyone who hires beneath the implicit quota is a discriminator, all of these possibilities are equilibria.

This type of multiplicity problem is an unfortunate result that stifles the predictive power of most signaling models. However, it can be argued that the out-of-equilibrium beliefs needed to construct the equilibria above are not empirically relevant. In particular, it may be unreasonable to assume that every deviation from a candidate equilibrium is a discriminator. Cho and Kreps (1987) posed an equilibrium refinement known as the "intuitive criterion." This criterion was constructed to aid in choosing among the multiplicity of possible equilibria found in most signaling games. The criterion is applied in my model in a series of steps.

1. For any deviation from a candidate equilibrium, define a set of types that would receive less than their equilibrium payoff by making the deviation, provided that the auditor plays an undominated strategy.
2. Define a set of types that would necessarily be better off by employing the deviating ratio, given that the auditor knows that the deviation would not be affected by any employer in the set defined in step 1.
3. If the set defined in step 2 is nonempty, the equilibrium fails the intuitive criterion.

In many (standard) signaling models, this refinement has eliminated the multiplicity problem. Cho and Kreps (1987) show, in the Spence (1973) model of job market signaling, that the intuitive criterion selects the separating equilibrium with the least amount of inefficient signaling.

Unfortunately, it has absolutely no bite in the current (nonstandard) model.⁸ Proposition 6 shows that all semiseparating equilibria (marginal and inclusive) survive after applying the intuitive criterion.

To see this, consider the three-step verification process outlined above. The proposition shows that the set of employers who strictly prefer their equilibrium payoff to any deviation is precisely the set of employers who are hiring their first-best ratio above the implicit quota. We know that an equilibrium fails the intuitive criterion if there exists a type who would necessarily be better off by deviating, given that the auditor will know that the deviation did not occur in any employer hiring its first-best ratio. However, there does not exist such a type because there is still a positive probability of being penalized even when an employer is not discriminating. In other words, imperfect information after the audit undermines the intuitive criterion.

3. CONCLUDING REMARKS

Many individuals have an allergic reaction to the use of quotas but seemingly want to eliminate discrimination by enforcing antidiscrimination laws. The main result in this paper shows that enforcing antidiscrimination policy has the unintended effect of causing all equilibria to involve a set of employers who alter their hiring ratios to avoid being audited, on account of the auditor's lack of information. In essence, goals are quotas whenever auditing technology is not perfect. And, under some assumptions, goals and targets can lead to more extreme quota-like hiring. Attacking affirmative action as a quota for minorities while endorsing goals and antidiscrimination enforcement is vacuous.

These results extend in natural and interesting ways to other realms of law and economics. For example, in a tax evasion model, the results indicate that there exist equilibria in which "honest" taxpayers are willing to overreport so that they are not fined. Future research in these areas can be extended along many dimensions. First, it would be interesting to construct a dynamic or repeated model of the auditing process in order to highlight the difficulties an auditor has in identifying discrimination in promotion policies relative to identifying discrimination in initial hiring. Auditors indicate that the former is much more difficult

8. This is also true for the stronger Dominance 1 (D1) and equilibrium dominance test, which requires one to believe, with probability one, that the type that deviated is the one who has the most incentive.

to monitor. A repeated-game model would have the advantage of employer reputations. Second, as mentioned in the text, one may want to endogenize the applicant pools in two dimensions: (1) allow workers to make human capital investments and (2) allow employers to invest in recruiting initiatives in hopes of being given a better pool of potential workers. Another viable extension might be a model in which workers can accuse their employer of discriminating. In fact, the U.S. Department of Labor has a discrimination complaint form on its Web page. Since most of the money collected by the Office of Federal Contract Compliance Programs is distributed among those workers who file the complaints, it may be interesting to examine the strategic relationships at play within this environment.

APPENDIX A: FORMAL MODEL AND PROOFS OF PROPOSITIONS

A1. The Basic Building Blocks

There is a continuum of workers and a continuum of employers, each with unit measure. There are two groups of workers: a measure λ are minorities, and $1 - \lambda$ are nonminorities. There are also two types of employers: a measure μ are biased against minorities, and a measure $1 - \mu$ are unbiased. There is also a large set of auditors. The government chooses a fine P before the start of the game.

Nature moves first and assigns a type (t, a) to each employer, where $t = b$ or $t = u$ if an employer is biased or unbiased, respectively, and distributes an applicant pool $a \in [g, \bar{a}]$ to each employer according to a smooth and continuous cumulative distribution function $F(a)$ and related density $f(a)$, where a represents the profit-maximizing (absent discriminatory taste) ratio of minorities to nonminorities in an employer's applicant pool. To avoid trivialities, I assume that every applicant pool has at least one minority candidate. Next, employers observe their two-dimensional private type, (t, a) , and make a workforce decision $r(t, a) \in [0, \infty)$.

By definition, $r(u, a) = a$; that is, absent regulation, unbiased employers hire the profit-maximizing ratio of minorities to nonminorities. When optimizing, biased employers will hire a ratio $(1 - \alpha)a$, $\alpha \in (0, 1)$, absent regulation. An auditor, after observing r (not t or a), makes a dichotomous audit decision: audit or not. If the auditor decides to conduct an audit, she makes a correct assessment of the employer with probability $\phi > \frac{1}{2}$, and she makes a mistake with probability $1 - \phi$. After the audit, the auditor makes a punishment decision, deciding whether or not to issue the fine P .

A2. Payoffs

Employers' payoffs are represented by a function $\Gamma[r(u, a) - a]$ for unbiased employers and a function $\Gamma[r(b, a) - (1 - \alpha)a]$ for biased employers. I make the following assumptions on $\Gamma(\cdot)$.

Assumption 1. The function $\Gamma(z)$ is twice continuously differentiable, strictly concave, and symmetric ($\Gamma(z) = \Gamma(-z)$) and achieves a maximum of zero when $z = 0$.

Let $\beta > 0$ denote the costs to the auditor of engaging in an audit of an employer. Auditors receive a payoff $\chi > \beta$ if they punish a discriminating employer and suffer a cost $-c < 0$ if they do not fine a discriminating employer or mistakenly fine a nondiscriminator. The payoff to an auditor for not punishing a nondiscriminator is normalized to zero. I assume that P , χ , c , and β are exogenously given.

A3. Strategies

A strategy for an employer is an assignment function that maps its private type (t, a) to a ratio of minorities to nonminorities hired. A strategy for an auditor is a function that maps an employer's observed ratio of minorities to nonminorities hired to an audit decision.

To begin, I restrict attention to monotonic strategies (Q) for the auditor (that is, cutoffs), in which she audits any employer with an observed ratio of minorities to nonminorities $r < Q$ and does not audit any employer with an observed ratio $r \geq Q$. This is without loss of generality when the auditor uses pure strategies (see Appendix B).

A4. Expected Payoffs

Let $\Psi(r)$ denote the probability that the employer is discriminating, conditional on hiring a workforce r .⁹ The auditor's expected payoff of not conducting an audit is $-\Psi(r)c$. When optimizing, the auditor believes that the employer is discriminating with probability $\Psi(r)\phi + [1 - \Psi(r)](1 - \phi)$ and punishes him with payoff $\Psi(r)\phi\chi - [1 - \Psi(r)](1 - \phi)c$. With probability $\Psi(r)(1 - \phi) + [1 - \Psi(r)]\phi$, she thinks that the employer is not discriminating and does not punish him (since she receives negative payoff for doing so). The auditor's expected payoff can be written as

$$\Psi(r)\phi\chi - (1 - \phi)c - \beta.$$

The employer's expected payoffs depend on his profit-maximizing hiring ratio, the auditor's cutoff strategy, and the expected cost of being audited. An unbiased employer's expected payoff of employing a workforce $r(u, a)$ is $\Gamma[r(u, a) - a] - \phi P \delta_{r < Q}$ if he discriminates and $\Gamma[r(u, a) - a] - (1 - \phi)P \delta_{r < Q}$ if he

9. The explicit derivation of $\Psi(r)$ will be equilibrium specific.

does not, where δ is a standard indicator function. Similarly, a biased employer's expected payoff of employing a workforce $r(b, a)$ is $\Gamma[r(b, a) - (1 - \alpha)a] - \phi P \delta_{r < Q}$ if he discriminates and $\Gamma[r(b, s) - (1 - \alpha)a] - (1 - \phi)P \delta_{r < Q}$ if he does not.

With this notation in hand, I can provide the values needed to recreate the numerical example discussed in the text: $\Psi(r) = 1 - r$, $\phi = \frac{4}{5}$, $\chi = c = 1$, and $\beta = \frac{7}{10}$. The resulting value of Q is $\frac{1}{2}$. Unbiased and biased employers are given by $\Gamma(r) = -(a - r)^2$ and $\Gamma(r) = -[(1 - \alpha)a - r]^2$, respectively; $\alpha = \frac{1}{2}$; $P = \frac{5}{64}$ for a marginal equilibrium and $P > \frac{5}{4}$ for an inclusive equilibrium; and $\hat{r} = \frac{3}{8}$.

Proposition 1. No pseudoseparating equilibrium exists.

Proof. To see that no pseudoseparating equilibrium exists, it is sufficient to show that at least one employer will have an incentive to deviate from its first-best ratio whenever $\phi < 1$. Consider the employer who has a profit-maximizing workforce with a ratio smaller than the auditor's threshold (Q). (We know this employer exists because of the continuity assumptions.) For an unbiased employer whose first-best ratio is slightly less than the expected auditing threshold, the following equation must hold:

$$\Gamma(0) - (1 - \phi)P > \Gamma(\varepsilon).$$

We know $\Gamma(0) = 0$ by assumption, so

$$-(1 - \phi)P > \Gamma(\varepsilon).$$

However, for any fixed $\phi P > 0$, there exists a value of ε small enough such that this inequality does not hold. Note that $Q = 0$ is ruled out by the definition of r^* . Q.E.D.

Next, I provide two definitions.

Definition 3. In a marginal semiseparating equilibrium, $r(u, a) = Q$ for all $a \in [\hat{a}, Q]$, $r(u, a) = a$ for all $a \in [0, \hat{a}] \cup [Q, \bar{a}]$, $r(b, a/(1 - \alpha)) = Q$ for all $a \in [\hat{a}/(1 - \alpha), Q/(1 - \alpha)]$, and $r(b, a/(1 - \alpha)) = a$ for all $a \in [0, \hat{a}] \cup [Q, \bar{a}]$.

Definition 4. In an inclusive semiseparating equilibrium, $r(u, a) = Q$ for all $a \in [0, Q]$, $r(u, a) = a$ for all $a \in [Q, \bar{a}]$, $r(b, a/(1 - \alpha)) = Q$ for all $a \in [0, Q/(1 - \alpha)]$, and $r(b, a/(1 - \alpha)) = a$ for all $a \in [Q, \bar{a}]$.

And let $r^* > 0$ satisfy

$$\Psi(r^*) = \frac{\mu f[r^*/(1 - \alpha)]}{\mu f[r^*/(1 - \alpha)] + (1 - \mu)f(r^*)} = \frac{\beta + (1 - \phi)c}{\chi\phi + c}.$$

In words, r^* is the smallest ratio r for which the auditor does not find it optimal to audit when employers are hiring their first-best ratios. The next result provides an existential result for a marginal equilibrium.

Proposition 2. A marginal equilibrium exists if and only if there exists an $\hat{r} \in (0, Q)$ such that the following conditions hold:

$$\frac{\phi}{1-\phi} = \frac{\Gamma[\hat{r}/(1-\alpha) - Q]}{\Gamma(\hat{r} - Q)} \quad \text{for some } \hat{r} \in [0, Q], \quad \Psi(Q) \leq \frac{\beta + (1-\phi)c}{\chi\phi + c},$$

and any deviation $r' \in [\hat{r}, Q]$ (out-of-equilibrium event) is thought to indicate a discriminator.

Proof. By definition, in any marginal semiseparating equilibrium, there exists a nonempty set of employers such that $r(t, a) < Q$.

Claim 1. In any semiseparating equilibrium, if $r(t, a) < Q$, $r(t, a) \in \{a(1-\alpha), a\}$.

Proof of Claim 1. Suppose $(t, a) = (b, a)$. In this case, he will choose r that satisfies

$$\max \left\{ \begin{array}{l} -\phi P, \\ \Gamma(\alpha a) - (1-\phi)P, \\ \Gamma[z - (1-\alpha)a] - (1-\phi)P \end{array} \right\} \quad \text{for some } z > a.$$

However, the second term is always larger than the third, so I can rewrite this as

$$\max \left\{ \begin{array}{l} -\phi P, \\ \Gamma(\alpha a) - (1-\phi)P \end{array} \right\}.$$

A similar argument shows that type (u, a) employers will hire a , which is the desired result for claim 1.

Then, there must exist a ratio $\hat{r} < Q$ (strict inequality follows directly from $\phi < 1$) such that any employer with $a < \hat{r}$ hires a and any employer with first-best ratio $a \in (\hat{r}, Q]$ hires Q . Thus, the employer with $a = \hat{r}$ must be indifferent between hiring his first-best ratio and hiring Q . Further, we know that for any ratio r , there exists a such that $r[b, a/(1-\alpha)] = r(u, a) = r$. Therefore, we know that at ratio \hat{r} , there is an unbiased employer in state \hat{a} and a biased employer in state $\hat{a}/(1-\alpha)$. To ensure that both employers are indifferent at \hat{r} , the following equations must hold:

$$-(1-\phi)P = \Gamma(Q - \hat{a})$$

for unbiased types, which implies that $P = -\Gamma(Q - \hat{a})/(1-\phi)$, and

$$-\phi P = \Gamma[Q - \hat{a}(1-\alpha)]$$

for biased types, which implies that $P = -\Gamma[Q - \hat{a}(1-\alpha)]/\phi$. Thus, if $\phi/(1-\phi) = \Gamma[Q - \hat{a}(1-\alpha)]/\Gamma(Q - \hat{a})$, both equations are satisfied simultaneously.

To make it optimal for the employer to audit below Q , but not audit at or above, it must be that $\hat{r} \leq r^*$,

$$\frac{\mu \int_{\hat{r}(Q)/(1-\alpha)}^{Q/(1-\alpha)} f(a) da}{\mu \int_{\hat{r}(Q)/(1-\alpha)}^{Q/(1-\alpha)} f(a) da + (1-\mu) \int_{\hat{r}(Q)}^Q f(a) da} \leq \frac{\beta + (1-\phi)c}{\chi\phi + c},$$

and $Q \geq r^*$, respectively. Q.E.D.

Proposition 3. An inclusive equilibrium exists if and only if the following conditions hold:

$$P > \max \left\{ \frac{-\Gamma(-Q)}{1-\phi}, \frac{-\Gamma(-Q)}{\phi} \right\}, \quad \Psi(Q) \leq \frac{\beta + (1-\phi)c}{\chi\phi + c},$$

and any deviation $r' \in [0, Q]$ (out-of-equilibrium event) is thought to indicate a discriminator.

Proof. Suppose, by way of contradiction, that there exists separation to the left of Q . Given claim 1 above, there must exist a ratio $\hat{r} < Q$ (strict inequality again follows directly from $\phi < 1$) such that any employer with $a < \hat{r}$ hires a and any employer with first-best ratio $a \in [\hat{r}, Q]$ hires Q . Thus, the employer with $a = \hat{r}$ must be indifferent between hiring his first-best ratio and hiring Q . Further, we know that for any ratio r , there exists a such that $r[b, a/(1-\alpha)] = r[u, a] = r$. Therefore, we know at ratio \hat{r} that there is an unbiased employer in state \hat{a} and a biased employer in state $\hat{a}/(1-\alpha)$. To ensure that both employers are indifferent at \hat{r} , the following equations must hold:

$$-(1-\phi)P = \Gamma(Q - \hat{a})$$

for unbiased types and

$$-\phi P = \Gamma[Q - \hat{a}(1-\alpha)]$$

for biased types. Given

$$\alpha > 0, \quad P > \max \left\{ \frac{\Gamma(a - Q)}{1-\phi}, \frac{\Gamma[a(1-\alpha) - Q]}{\phi} \right\},$$

both equations cannot be satisfied simultaneously. Therefore, we need \hat{r}_b or \hat{r}_u for biased or unbiased types, respectively, where $\hat{r}_b < \hat{r}_u$, which satisfies

$$P = -\frac{\Gamma(Q - \hat{r}_u)}{(1-\phi)}$$

and

$$P = -\frac{\Gamma(Q - \hat{r}_b)}{\phi}.$$

Thus,

$$\frac{\phi}{1-\phi} = \frac{\Gamma(\underline{Q} - \hat{r}_b)}{\Gamma(\underline{Q} - \hat{r}_u)},$$

which is a contradiction (the right-hand side is greater than one, and the left-hand side is less than one, by definition). Q.E.D.

Let

$$g(r) \equiv \frac{\mu f[r/(1-\alpha)]}{\mu f[r/(1-\alpha)] + (1-\mu)f(r)}$$

denote the probability that a profit-maximizing ratio r is hired by a biased employer, where μ denotes the fraction of biased employers in the labor market. I assume that $g'(r) < 0$.¹⁰

Proposition 4. There exists a vector (\underline{Q}, P) such that for any $Q \geq \underline{Q}$, an inclusive semiseparating equilibrium exists if all $r' < Q$ (out-of-equilibrium events) are deemed discriminators.

Proof. Let

$$\underline{Q} \equiv \min \left\{ Q : \frac{\mu_Q Q / (1-\alpha) f(a) da}{\mu_Q Q / (1-\alpha) f(a) da + (1-\mu_Q) \int_0^Q f(a) da} \leq \frac{\beta + (1-\phi)c}{\chi\phi + c} \right\}.$$

It follows directly from proposition 3 that for any $Q \geq \underline{Q}$, the conditions of the proposition are met if deviators below Q are thought to be discriminators and $(1-\phi)P$ is sufficiently high. Now, it suffices to show that

$$\frac{\mu_Q \int_0^{Q/(1-\alpha)} f(a) da}{\mu_Q \int_0^{Q/(1-\alpha)} f(a) da + (1-\mu_Q) \int_0^Q f(a) da}$$

is decreasing in Q , for $Q \in [\max\{r^*, \underline{Q}\}, \bar{a}]$. Since

$$\Psi(Q) = \frac{\mu_Q \int_0^{Q/(1-\alpha)} f(a) da}{\mu_Q \int_0^{Q/(1-\alpha)} f(a) da + (1-\mu_Q) \int_0^Q f(a) da},$$

we have the desired result. Q.E.D.

Proposition 5. There exists a vector $(\underline{Q}, P, \hat{r})$ such that for any $Q \geq \underline{Q}$, a marginal semiseparating equilibrium exists if all employers who choose $r' \in [\hat{r}, Q]$ (out-of-equilibrium event) are deemed discriminators.

Proof. Recall that, in equilibrium, $\hat{r}(Q)$. The rest follows directly from proposition 4. Q.E.D.

10. Taking the first-order derivative, $g' < 0$ if and only if $[f'(r)]/[f(r)] > \{f'[r/(1-\alpha)]\}/\{(1-\alpha)f[r/(1-\alpha)]\}$. This is the same condition as $\ln f(r)$ decreasing in r , which is consistent with many distributional assumptions on f .

Proposition 6. All semiseparating equilibria satisfy the intuitive criterion.

Proof. Using the notation found in Fudenberg and Tirole (1991, p. 448), let $\Theta = \{u, b\} \times [0, \bar{a}]$ denote the set of types, with any particular type denoted $\theta = (t, a)$; $A \in \{0, 1\}$ denote the auditor's choice variable, where $A = 1$ if she decides to audit; and ξ denote the auditor's beliefs and $u(r', A, \theta)$ the auditor's payoff. Now, define the set of auditor best responses as

$$\text{BR}(\Theta, r') = \bigcup_{\xi: \xi(\Theta|r)=1} \text{BR}(\xi, r'),$$

where

$$\text{BR}(\xi, r') = \arg \max_A \int_{\theta \in \Theta} \xi(\theta|r') u(r', A, \theta).$$

Let $u_E^*(\theta)$ denote the equilibrium payoff to a type θ employer. Define a set

$$J(r') = \{\theta : u_E^*(\theta) > \max_{A \in \text{BR}(\Theta, r')} u_E(r', A, \theta)\}.$$

It is straightforward to see that this set consists of all values of θ such that $r_\theta^* \geq Q$. Therefore, rewrite $J(r')$ as

$$J(r') = \{\theta : r_\theta^* \geq Q\}.$$

By definition, the equilibrium fails the intuitive criterion if for some values of r' there exists a $\theta' \in \Theta$ such that

$$u_E^*(\theta') < \min_{A \in \text{BR}(\Theta/J(r'), r')} u_E(r', A, \theta').$$

However, since the set $\Theta/J(r')$ contains discriminators and $\text{BR}[\Theta/J(r'), r']$ contains $A = 1$, it follows that

$$\{\theta' \in \Theta : u_E^*(\theta') < \min_{A \in \text{BR}(\Theta/J(r'), r')} u_E(r', A, \theta')\} = \emptyset,$$

which is the desired result. A virtually identical argument proves the analogous result for the set of marginal semiseparating equilibria. Q.E.D.

APPENDIX B: ADDITIONAL CALCULATIONS

B1. Pooling Equilibria

In a pooling equilibrium, all types choose the same action. In particular, a pooling equilibrium exists at r_p if for all $(t, a) \in \{b, u\} \times [0, \bar{a}]$, $r(t, a) = r_p$. In what follows, I prove the existence of a unique pooling equilibrium for my general model.

Proposition 7. If $P > \Gamma(\bar{a})/(1 - \phi)$, a unique pooling equilibrium exists at $r_p = \bar{a}$, provided that the auditor believes all employers who hire $r < r_p$ are discriminating.

Proof. Suppose that for all (t, a) , $r(t, a) = \bar{a}$, and that the auditor does not audit any employer with hiring ratio \bar{a} but audits any employer with $r < \bar{a}$. The auditor has no incentive to deviate from this strategy, since she knows that there is no possibility that anyone with $r = \bar{a}$ is discriminating, and it is consistent for her to believe that any deviators are discriminating. For employers, no one has incentive to deviate, since they will be audited and $P > \Gamma(\bar{a})/(1 - \phi)$. To establish uniqueness, suppose, by way of contradiction, that there exists a pooling equilibrium at $r_p < \bar{a}$. In this case, (t, a) , $r(t, a) = r_p$ for all. However, this implies that even an unbiased employer in state \bar{a} finds it best to deviate. In symbols, this requires that $0 < \Gamma(r_p - \bar{a})$, since the auditor will not find it optimal to investigate any employer with $r = \bar{a}$. This contradicts the assumptions on $\Gamma(\cdot)$. Q.E.D.

This proposition provides a knife-edge possibility for the existence of pooling equilibria. The result seems innocuous due to the fact that I do not allow for reverse discrimination in my simple model. This would alleviate such an extreme pooling equilibrium, although I am not certain whether it could guarantee the nonexistence of less extreme pooling equilibria.

B2. Nonmonotonic Equilibria

In my analysis thus far, I have restricted my attention to monotonic auditing strategies. In this section, I relax that assumption and analyze the existence of equilibria in which the auditor sets nonmonotonic threshold strategies.¹¹ These strategies involve multiple auditing thresholds. This implies that the auditor believes there are certain minority/nonminority hiring ratios that are “just right”: anything too low or too high is suspect. The final technical result shows that nonmonotonic equilibria do not exist in pure strategies.

Proposition 8. No nonmonotonic equilibrium exists.

Proof. In any nonmonotonic equilibrium, there exist at least two auditorial thresholds Q_1 and Q_2 , where it is assumed, without loss of generality, that $Q_1 < Q_2$. Then for all $r > Q_2$ the auditor wants to audit, and for all $r \leq Q_2$ she does not (otherwise one could assume one auditing threshold without loss). Q.E.D.

We know that $Q_2 \in (Q_1, \bar{a}]$. However, it can be shown that $Q_2 = \bar{a}$. Suppose not. Then any employer whose first-best hiring ratio $a \geq Q_2$ hires Q_2 . However, this can never be optimal for the type (u, \bar{a}) employer, given that he is guaranteed not to be audited if he hires his first-best ratio. Therefore, the only possible nonmonotonic equilibria requires $Q_1 < Q_2 = \bar{a}$. I rule this case out a priori, given that this boils down to the auditor using a monotonic strategy (since she must audit only an employer with a hiring ratio greater than \bar{a}).

11 Technically speaking, these equilibria are in the set of semiseparating equilibria.

B3. Mixed-Strategy Equilibria

For the auditor to play a mixed strategy between multiple thresholds, she must be indifferent between auditing and not at these thresholds. Therefore, $\Psi(r) = [\beta + (1 - \phi)c]/(\chi\phi + c)$ must have multiple solutions. For tractability, let

$$\Xi \equiv \left\{ r : \Psi(r) = \frac{\beta + (1 - \phi)c}{\chi\phi + c} \right\}$$

denote the set of such solutions with its cardinality $|\Xi| \in [1, \infty]$. Finally, let ρ_j denote the probability that an auditor audits at threshold Q_j , where we assume without loss that $Q_1 < Q_2 < \dots < Q_{\#\Xi}$.

The employer's problem is straightforward. A type (t, a) employer hires his first-best ratio if that hiring ratio is above $Q_{\#\Xi}$; employers whose profit-maximizing hiring ratio is below $Q_{\#\Xi}$ choose

$$\max \left\{ \begin{array}{c} -(1 - \phi)P, \\ \Gamma(Q_1 - a)(1 - \rho_1)(1 - \phi)P, \\ \Gamma(Q_2 - a)(1 - \rho_2)(1 - \phi)P, \\ \Gamma(Q_{\#\Xi} - a)(1 - \rho_{\#\Xi})(1 - \phi)P, \\ \dots, \\ \Gamma(Q_{\#\Xi} - s) \end{array} \right\}$$

if unbiased and

$$\max \left\{ \begin{array}{c} -\phi P, \\ \Gamma(Q_1 - (1 - \alpha)a) - (1 - \rho_1)\phi P, \\ \Gamma(Q_1 - a) - (1 - \rho_1)(1 - \phi)P, \\ \dots \end{array} \right\}$$

if biased.

For the auditor, it must be that $\Psi(Q_j^*) = [\beta + (1 - \phi)c]/(\chi\phi + c)$ for all $Q_j^* \in \Xi$. Given the general framework for mixed-strategy equilibria, one can check mixed strategies for particular parameter values as needed.

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